Relational Algebraic Equivalence Transformation Rules

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections; *cascade of* σ.

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. Selection operations are commutative:

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the final operations in a sequence of projection operations is needed, the others can be omitted; *cascade of* Π

$$\Pi_{L_1}(\Pi_{L_2}(\ldots(\Pi_{L_n}(E))\ldots)) = \Pi_{L_1}(E)$$

4. Selections can be combined with Cartesian products and theta joins:

$$\sigma_{\theta}(E_1 \times E_2) = E_1 \bowtie_{\theta} E_2$$

$$\sigma_{\theta_1}(E_1 \bowtie_{\sigma_{\theta_2}} E_2) = E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$$

5. Theta join operations are commutative:

$$E_1 \bowtie_{\theta} E_2 = E_2 \bowtie_{\theta} E_1$$

6. Natural-join operations are associative:

$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

   Theta joins are associative in the following manner

$$(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

   where θ_2 involves attributes from E_2 and E_3 only.

7. The selection operation distributes over the theta join operation under the following two conditions:

   (a) It distributes when all the attributes in the selection condition θ_0 involve only the attributes of one of the expressions (E_1) being joined.

   $$\sigma_{\theta_0}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_0}(E_1)) \bowtie_\theta E_2$$

   (b) It distributes when the selection condition θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2

   $$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_\theta (\sigma_{\theta_2}(E_2))$$

8. The projection operation distributes over the theta join.

   (a) Let L_1 and L_2 be attributes of E_1 and E_2 respectively. Suppose that the join condition θ involves only attributes in $L_1 \cup L_2$. Then

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = (\Pi_{L_1}(E_1)) \bowtie_\theta (\Pi_{L_2}(E_2))$$

   (b) Consider a join $E_1 \bowtie_\theta E_2$. Let L_1 and L_2 be sets of attributes from E_1 and E_2 respectively. Let L_3 be attributes of E_1 that are involved in the join condition θ, but are not in $L_1 \cup L_2$, and let L_4 be attributes of E_2 that are involved in the join condition θ, but are not in $L_1 \cup L_2$. Then

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1 \cup L_2}((\Pi_{L_1 \cup L_3}(E_1)) \bowtie_\theta (\Pi_{L_2 \cup L_4}(E_2)))$$

9. The set operations union and intersection are commutative.

   $$E_1 \cup E_2 = E_2 \cup E_1$$

   $$E_1 \cap E_2 = E_2 \cap E_1$$

   Set difference is not commutative.

10. Set union and intersection are associative.

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$$

$$(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

11. The selection operation distributes over the union, intersection, and set-difference operations.

$$\sigma_P(E_1 - E_2) = \sigma_P(E_1) - E_2 = \sigma_P(E_1) - \sigma_P(E_2)$$

12. The projection operation distributes over the union operation.

$$\Pi_L(E_1 \cup E_2) = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$